Archive of Applied Mechanics 74 (2004) 288–298 © Springer-Verlag 2004
DOI 10.1007/s00419-004-0350-9

Amplification of acoustic waves in laminated piezoelectric semiconductor plates

J. S. Yang, X. M. Yang, J. A. Turner

Summary Two-dimensional equations for coupled extensional, flexural and thickness-shear motions of laminated plates of piezoelectric semiconductors are obtained systematically from the three-dimensional equations by retaining lower order terms in power series expansions in the plate thickness coordinate. The equations are used to analyze extensional waves in a composite plate of piezoelectric ceramics and semiconductors. Dispersion and dissipation due to semiconduction as well as wave amplification by a dc electric field are discussed.

Keywords piezoelectricity, semiconductor, plate, composite

1
Introduction

Piezoelectric materials can be either dielectrics or semiconductors [1]. An acoustic wave propagating in a piezoelectric crystal is usually accompanied by an electric field. When the crystal is also semiconducting, the field produces currents and space charge resulting in dispersion and acoustic loss [2]. The interaction between a traveling acoustic wave and mobile charges in piezoelectric semiconductors is called the acoustoelectric effect which is a special case of a more general phenomenon which may be called wave-particle drag [3]. It was also found that an acoustic wave traveling in a piezoelectric semiconductor can be amplified by application of a dc electric field [4]. Acoustoelectric effect and acoustoelectric amplification of acoustic waves have led to the development of acoustoelectric devices [5–8]. The basic behavior of piezoelectric semiconductors and acoustoelectric effect can be described by a linear phenomenological theory [2, 4]. More sophisticated nonlinear theories for deformable semiconductors have also been developed [9, 10]. Acoustoelectric effect can also be produced in composites of piezoelectric insulators and nonpiezoelectric semiconductors [8]. In these composites the acoustoelectric effect is due to the combination of the piezoelectric effect and semiconduction in each component phase of the composite. Laminated plates of piezoelectric insulators and nonpiezoelectric semiconductors can be used to produce acoustoelectric effect for device applications. Due to anisotropy and field coupling, modeling of plate piezoelectric devices, single and/or multi-layered, is usually very challenging. Exact analysis from three-dimensional equations is possible in idealized special cases. Two-dimensional structural theories for thin piezoelectric insulator plates have been developed [11–13] and proved very effective in real device modeling as discussed by Wang and Yang [14]. In this paper we study motions of thin plates of laminated piezoelectric semiconductors. The three-dimensional equations of linear piezoelectric semiconductors are summarized in Sect. 2. Two-dimensional equations for laminated plates are derived systematically from the three-dimensional equations in Sect. 3 and are truncated to a first-order theory for coupled extension, flexure and thickness-shear in Sect. 4. Propagation of extensional waves under a dc field is analyzed in Sect. 5. Finally, some conclusions are drawn in Sect. 6.

Received 12 February 2004; accepted for publication 2 May 2004

J. S. Yang (✉), X. M. Yang, J. A. Turner
Department of Engineering Mechanics
University of Nebraska, Lincoln, NE 68588-0526, USA
Tel.: +1-402-472-0712
Fax: +1-402-472-8292
e-mail: jyang1@unl.edu

2
Three-dimensional equations

Consider a homogeneous, one-carrier piezoelectric semiconductor under a uniform dc electric field $\bar{E}_j$. The steady state current is $\bar{J}_i = q\bar{n}\mu_{ij}\bar{E}_j$, where q is the carrier charge which may be the electronic charge or its negative, $\bar{n}$ is the steady state carrier density which produces electrical neutrality, and μ_{ij} is the carrier mobility. The summation convention for repeated tensor indices is used. When an acoustic wave propagates through the material, perturbations of the electric field, the carrier density and the current are denoted by E_j, n and J_i. The linear theory for small signals consists of the equations of motion, Gauss's law of electrostatics, and conservation of charge [2, 4, 15]

$$T_{ji,j} = \rho\ddot{u}_i, \quad D_{i,i} = qn, \quad q\dot{n} + J_{i,i} = 0, \tag{1}$$

where u_i is the displacement vector, T_{ij} the stress tensor, ρ the mass density, and D_i the electric displacement vector. A comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index. A superimposed dot represents differentiation with respect to time t. The above equations are accompanied by the following constitutive relations

$$\begin{aligned} T_{ij} &= c_{ijkl}S_{kl} - e_{kij}E_k, \\ D_i &= e_{ijk}S_{jk} + \varepsilon_{ij}E_j, \\ J_i &= q\bar{n}\mu_{ij}E_j + qn\mu_{ij}\bar{E}_j - qd_{ij}N_j, \end{aligned} \tag{2}$$

where the strain tensor S_{ij}, the electric potential ϕ, and the carrier density gradient N_j are defined by

$$S_{ij} = (u_{i,j} + u_{j,i})/2, \quad E_i = -\phi_{,i}, \quad N_j = n_{,j}. \tag{3}$$

In Eq. (2), c_{ijkl}, e_{kij} and ϵ_{ij} are the elastic, piezoelectric and dielectric constants, respectively, while d_{ij} are the carrier diffusion constants. With successive substitutions from Eqs. (2) and (3), Eq. (1) can be rewritten for $\mathbf{u}$, ϕ and n as

$$\begin{aligned} &c_{ijkl}u_{k,lj} + e_{kij}\phi_{,kj} = \rho\ddot{u}_i, \\ &e_{ikl}u_{k,li} - \varepsilon_{ij}\phi_{,ij} = qn, \\ &\dot{n} - \bar{n}\mu_{ij}\phi_{,ij} + \mu_{ij}\bar{E}_j n_{,i} - d_{ij}n_{,ij} = 0. \end{aligned} \tag{4}$$

On the boundary of a finite body with a unit outward normal n_i, usually the mechanical displacement u_i or the traction vector $T_{ij}n_i$, the electric potential ϕ or the normal component of the electric displacement vector $D_i\, n_i$, and the carrier density n or the normal current $J_i n_i$ are known.

3
Series of two-dimensional plate equations

Consider an N-layer laminated piezoelectric semiconductor plate of thickness $2h$ with the x_3 axis normal to the plate. The two plate faces and $N-1$ interfaces are sequentially determined by $x_3 = -h = h_0, h_1, \ldots, h_{N-1}$, and $h_N = h$. First we expand the mechanical displacement, electric potential and carrier density into power series in x_3 in the manner of [11]

$$u_i = \sum_n x_3^n u_i^{(n)}, \quad \phi = \sum_n x_3^n \phi^{(n)}, \quad n = \sum_n x_3^n n^{(n)}, \tag{5}$$

where $u_i^{(n)}$, $\phi^{(n)}$ and $n^{(n)}$ $(n = 0, 1, \ldots, N)$ are functions of x_1, x_2 and t only. From Eq. (3) we have

$$S_{ij} = \sum_n x_3^n S_{ij}^{(n)}, \quad E_i = \sum_n x_3^n E_i^{(n)}, \quad N_j = \sum_n x_3^n N_j^{(n)}, \tag{6}$$

where

$$
\begin{aligned}
S_{ij}^{(n)} &= \frac{1}{2}[u_{j,i}^{(n)} + u_{i,j}^{(n)} + (n+1)(\delta_{i3}u_j^{(n+1)} + \delta_{3j}u_i^{(n+1)})],\\
E_i^{(n)} &= -\phi_{,i}^{(n)} - (n+1)\delta_{3i}\phi^{(n+1)},\\
N_j^{(n)} &= n_{,j}^{(n)} + (n+1)\delta_{3j}n^{(n+1)}.
\end{aligned} \tag{7}
$$

Then we multiply Eq. (1) by x_3^n, integrate the resulting equations across the thickness of the I-th layer from h_{I-1} to h_I, sum over I, and make use of the interface continuity conditions of the traction vector, the normal component of the electric displacement, and the normal component of the current. We then obtain the following n-th order field equations

$$
\begin{aligned}
&T_{ij,i}^{(n)} - nT_{3j}^{(n-1)} + t_j^{(n)} = \sum_m \rho^{(m+n)}\ddot{u}_j^{(m)},\\
&D_{i,i}^{(n)} - nD_3^{(n-1)} + d^{(n)} = \sum_m q^{(m+n)}n^{(n)},\\
&\sum_m q^{(m+n)}\dot{n}^{(n)} + J_{i,i}^{(n)} - nJ_3^{(n-1)} + j^{(n)} = 0,
\end{aligned} \tag{8}
$$

where the plate resultants, surface loads, mass and charge are defined by

$$
\begin{aligned}
&\{T_{ij}^{(n)}, D_i^{(n)}, J_i^{(n)}\} = \int_{-h}^{h} x_3^n\{T_{ij}, D_i, J_i\}dx_3 = \sum_{I=1}^{N}\int_{h_{I-1}}^{h_I} x_3^n\{T_{ij}, D_i, J_i\}dx_3,\\
&t_j^{(n)} = [x_3^n T_{3j}]_{-h}^{h}, \quad d^{(n)} = [x_3^n D_3]_{-h}^{h}, \quad j^{(n)} = [x_3^n J_3]_{-h}^{h},\\
&\rho^{(m+n)} = \sum_{I=1}^{N}(\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3)\rho^I, \quad q^{(m+n)} = \sum_{I=1}^{N}(\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3)q^I.
\end{aligned} \tag{9}
$$

Next we derive plate constitutive equations. Since the plate is assumed to be thin, we make the stress relaxation approximation of vanishing normal stress $T_{33} = 0$. This implies that by setting $i = j = 3$ in the first part of Eq. 2, the following expression results for $u_{3,3}$ in terms of other components of the displacement and potential gradients

$$
u_{3,3} = -\frac{1}{c_{3333}}(c_{33kl}u_{k,l} - c_{3333}u_{3,3} - e_{k33}E_k). \tag{10}
$$

Stress relaxation for thin anisotropic or piezoelectric plates can be made in ways more sophisticated than the above, also involving T_{31} and T_{32} [11]. That is not our main interest here. The above relaxation involving T_{33} is the major relaxation because in anisotropic plates couplings among extensions in different directions are much stronger than couplings between extensions and shears. In Eq. (10), $u_{3,3}$ has been eliminated on the right hand side because when $i = j = 3$ the two terms containing $u_{3,3}$ will cancel each other. From Eq. (10) the thickness expansion or contraction accompanying the extension and flexure of the plate due to Poisson's effect can be found. Substituting Eq. (10) into the first two parts of Eq. (2), we obtain the following constitutive relations relaxed for thin plates

$$
\begin{aligned}
T_{ij} &= \bar{c}_{ijkl}u_{k,l} - \bar{e}_{kij}E_k,\\
D_i &= \bar{e}_{ikl}u_{k,l} + \bar{\varepsilon}_{ij}E_j,
\end{aligned} \tag{11}
$$

where the relaxed material constants are defined by

$$
\begin{aligned}
\bar{c}_{ijkl} &= c_{ijkl} - c_{ij33}c_{33kl}/c_{3333},\\
\bar{e}_{kij} &= e_{kij} - e_{k33}c_{33ij}/c_{3333},\\
\bar{\varepsilon}_{ij} &= \varepsilon_{ij} + e_{i33}e_{j33}/c_{3333}.
\end{aligned} \tag{12}
$$

We note that the right hand sides of Eq. (11) do not contain $u_{3,3}$ and $T_{33} = 0$ is automatically satisfied. Substitution of Eq. (11) into Eq. (9) part one gives the constitutive equations of order n as follows

$$T_{ij}^{(n)} = \sum_m (c_{ijkl}^{(m+n)} S_{kl}^{(m)} - e_{kij}^{(m+n)} E_k^{(m)}),$$
$$D_i^{(n)} = \sum_m (e_{ijk}^{(m+n)} S_{jk}^{(m)} - \varepsilon_{ij}^{(m+n)} E_j^{(m)}), \tag{13}$$
$$J_i^{(n)} = \sum_m (\mu_{ij}^{(m+n)} E_j^{(m)} - d_{ij}^{(m+n)} N_j^{(m)} + \lambda_i^{(m+n)} n^{(m)}),$$

where

$$c_{ijkl}^{(m+n)} = \sum_{I=1}^{N} (\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3) \bar{c}_{ijkl}^I, \quad e_{kij}^{(m+n)} = \sum_{I=1}^{N} (\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3) \bar{e}_{kij}^I,$$
$$\varepsilon_{ij}^{(m+n)} = \sum_{I=1}^{N} (\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3) \bar{\varepsilon}_{ij}^I, \quad \mu_{ij}^{(m+n)} = \sum_{I=1}^{N} (\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3) q^I \bar{n}^I \mu_{ij}^I, \tag{14}$$
$$d_{ij}^{(m+n)} = \sum_{I=1}^{N} (\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3) q^I d_{ij}^I, \quad \lambda_i^{(m+n)} = \sum_{I=1}^{N} (\int_{h_{I-1}}^{h_I} x_3^n x_3^m dx_3) q^I \mu_{ij}^I \bar{E}_j^I.$$

4
First-order plate theory

For a first-order plate theory of coupled extensional, flexural and thickness-shear motions we make the following truncation of Eq. (5)

$$u_a(x_1, x_2, x_3, t) \cong u_a^{(0)}(x_1, x_2, t) + x_3 u_a^{(1)}(x_1, x_2, t), a = 1, 2,$$
$$u_3(x_1, x_2, x_3, t) \cong u_3^{(0)}(x_1, x_2, t) + x_3 u_3^{(1)}(x_1, x_2, t) + x_3^2 u_3^{(2)}(x_1, x_2, t), \tag{15}$$
$$\phi(x_1, x_2, x_3, t) \cong \phi^{(0)}(x_1, x_2, t) + x_3 \phi^{(1)}(x_1, x_2, t),$$
$$n(x_1, x_2, x_3, t) \cong n^{(0)}(x_1, x_2, t) + x_3 n^{(1)}(x_1, x_2, t),$$

where we have introduced a convention that the indices a (and later b) assume the values of 1 and 2 only but not 3. $u_a^{(0)}$ are the plate extensional displacements, $u_3^{(0)}$ the flexural displacement, and $u_a^{(1)}$ the thickness-shear displacements. $u_3^{(1)}$ and $u_3^{(2)}$ are the plate thickness-stretch displacements accompanying extension and flexure, which will be eliminated by the stress relaxation condition. The strains, electric field and carrier density gradient are, respectively,

$$S_p \cong S_p^{(0)} + x_3 S_p^{(1)}, \quad p = 1, 2, \cdots, 6,$$
$$E_i \cong E_i^{(0)} + x_3 E_i^{(1)}, \tag{16}$$
$$N_i \cong N_i^{(0)} + x_3 N_i^{(1)},$$

where, under the compact matrix notation [16], the indices p (and later q) range from 1 to 6. The zero- and first-order strains are

$$S_1^{(0)} = u_{1,1}^{(0)}, \quad S_2^{(0)} = u_{2,2}^{(0)}, \quad S_3^{(0)} = u_3^{(1)},$$
$$S_4^{(0)} = u_{3,2}^{(0)} + u_2^{(1)}, \quad S_5^{(0)} = u_{3,1}^{(0)} + u_1^{(1)}, \quad S_6^{(0)} = u_{1,2}^{(0)} + u_{2,1}^{(0)}, \tag{17}$$

and

$$S_1^{(1)} = u_{1,1}^{(1)}, \quad S_2^{(1)} = u_{2,2}^{(1)}, \quad S_3^{(1)} = 2u_3^{(2)},$$
$$S_4^{(1)} = 0, \quad S_5^{(1)} = 0, \quad S_6^{(1)} = u_{1,2}^{(1)} + u_{2,1}^{(1)}. \tag{18}$$

We note that $S_3^{(0)}$ and $S_3^{(1)}$ are related to $u_3^{(1)}$ and $u_3^{(2)}$, but they will not appear in the final constitutive relations because of Eqs. (11) and (12). The zero- and first-order electric fields are given by

$$E_1^{(0)} = -\phi_{,1}^{(0)}, \quad E_2^{(0)} = -\phi_{,2}^{(0)}, \quad E_3^{(0)} = -\phi^{(1)}, \tag{19}$$

and

$$E_1^{(1)} = -\phi_{,1}^{(1)}, \quad E_2^{(1)} = -\phi_{,2}^{(1)}, \quad E_3^{(1)} = 0. \tag{20}$$

The zero- and first-order gradients of the carrier density are

$$N_1^{(0)} = n_{,1}^{(0)}, \quad N_2^{(0)} = n_{,2}^{(0)}, \quad N_3^{(0)} = n^{(1)}, \tag{21}$$

and

$$N_1^{(1)} = n_{,1}^{(1)}, \quad N_2^{(1)} = n_{,2}^{(1)}, \quad N_3^{(1)} = 0. \tag{22}$$

The zero- and first-order equations of motion, Gauss's law and the conservation of charge take the following form

$$\begin{aligned}
&T_{ab,a}^{(0)} + t_b^{(0)} = \sum_{I=1}^{N} \rho^I [(h_I - h_{I-1}) \ddot{u}_b^{(0)} + \frac{h_I^2 - h_{I-1}^2}{2} \ddot{u}_b^{(1)}], \\
&T_{a3,a}^{(0)} + t_3^{(0)} = \sum_{I=1}^{N} \rho^I [(h_I - h_{I-1}) \ddot{u}_3^{(0)}], \\
&T_{ab,a}^{(1)} - T_{3b}^{(0)} + t_b^{(1)} = \sum_{I=1}^{N} \rho^I [\frac{h_I^2 - h_{I-1}^2}{2} \ddot{u}_b^{(0)} + \frac{h_I^3 - h_{I-1}^3}{3} \ddot{u}_b^{(1)}], \\
&D_{a,a}^{(0)} + d^{(0)} = \sum_{I=1}^{N} q^I [(h_I - h_{I-1}) n^{(0)} + \frac{h_I^2 - h_{I-1}^2}{2} n^{(1)}], \\
&D_{a,a}^{(1)} - D_3^{(0)} + d^{(1)} = \sum_{I=1}^{N} q^I [\frac{h_I^2 - h_{I-1}^2}{2} n^{(0)} + \frac{h_I^3 - h_{I-1}^3}{3} n^{(1)}], \\
&\sum_{I=1}^{N} q^I [(h_I - h_{I-1}) \dot{n}^{(0)} + \frac{h_I^2 - h_{I-1}^2}{2} \dot{n}^{(1)}] + J_{a,a}^{(0)} + j^{(0)} = 0, \\
&\sum_{I=1}^{N} q^I [\frac{h_I^2 - h_{I-1}^2}{2} \dot{n}^{(0)} + \frac{h_I^3 - h_{I-1}^3}{3} \dot{n}^{(1)}] + J_{a,a}^{(1)} - J_3^{(0)} + j^{(1)} = 0.
\end{aligned} \tag{23}$$

The zero-order plate constitutive relations are

$$\begin{aligned}
&T_{ij}^{(0)} = (c_{ijkl}^{(0)} S_{kl}^{(0)} - e_{kij}^{(0)} E_k^{(0)}) + (c_{ijkl}^{(1)} S_{kl}^{(1)} - e_{kij}^{(1)} E_k^{(1)}), \\
&D_i^{(0)} = (e_{ijk}^{(0)} S_{jk}^{(0)} + \varepsilon_{ij}^{(0)} E_j^{(0)}) + (e_{ijk}^{(1)} S_{jk}^{(1)} + \varepsilon_{ij}^{(1)} E_j^{(1)}), \\
&J_i^{(0)} = (\mu_{ij}^{(0)} E_j^{(0)} + \lambda_i^{(0)} n^{(0)} - d_{ij}^{(0)} N_j^{(0)}) + (\mu_{ij}^{(1)} E_j^{(1)} + \lambda_i^{(1)} n^{(1)} - d_{ij}^{(1)} N_j^{(1)}),
\end{aligned} \tag{24}$$

where

$$
\begin{aligned}
c_{ijkl}^{(0)} &= \sum_{I=1}^{N} (h_I - h_{I-1})\bar{c}_{ijkl}^{I}, \quad e_{kij}^{(0)} = \sum_{I=1}^{N} (h_I - h_{I-1})\bar{e}_{kij}^{I}, \\
\varepsilon_{ij}^{(0)} &= \sum_{I=1}^{N} (h_I - h_{I-1})\bar{\varepsilon}_{ij}^{I}, \quad \mu_{ij}^{(0)} = \sum_{I=1}^{N} (h_I - h_{I-1})q^I \bar{n}^I \mu_{ij}^{I}, \\
d_{ij}^{(0)} &= \sum_{I=1}^{N} (h_I - h_{I-1})q^I d_{ij}^{I}, \quad \lambda_{i}^{(0)} = \sum_{I=1}^{N} (h_I - h_{I-1})q^I \mu_{ij}^{I} \bar{E}_{j}^{I},
\end{aligned}
\tag{25}
$$

and

$$
\begin{aligned}
c_{ijkl}^{(1)} &= \sum_{I=1}^{N} (\frac{h_I^2 - h_{I-1}^2}{2})\bar{c}_{ijkl}^{I}, \quad e_{kij}^{(1)} = \sum_{I=1}^{N} (\frac{h_I^2 - h_{I-1}^2}{2})\bar{e}_{kij}^{I}, \\
\varepsilon_{ij}^{(1)} &= \sum_{I=1}^{N} (\frac{h_I^2 - h_{I-1}^2}{2})\bar{\varepsilon}_{ij}^{I}, \quad \mu_{ij}^{(1)} = \sum_{I=1}^{N} (\frac{h_I^2 - h_{I-1}^2}{2})q^I \bar{n}^I \mu_{ij}^{I}, \\
d_{ij}^{(1)} &= \sum_{I=1}^{N} (\frac{h_I^2 - h_{I-1}^2}{2})q^I d_{ij}^{I}, \quad \lambda_{i}^{(1)} = \sum_{I=1}^{N} (\frac{h_I^2 - h_{I-1}^2}{2})q^I \mu_{ij}^{I} \bar{E}_{j}^{I}.
\end{aligned}
\tag{26}
$$

The first-order constitutive relations are

$$
\begin{aligned}
T_{ij}^{(1)} &= (c_{ijkl}^{(1)} S_{kl}^{(0)} - e_{kij}^{(1)} E_{k}^{(0)}) + (c_{ijkl}^{(2)} S_{kl}^{(1)} - e_{kij}^{(2)} E_{k}^{(1)}), \\
D_{i}^{(1)} &= (e_{ijk}^{(1)} S_{jk}^{(0)} + \varepsilon_{ij}^{(1)} E_{j}^{(0)}) + (e_{ijk}^{(2)} S_{jk}^{(1)} + \varepsilon_{ij}^{(2)} E_{j}^{(1)}), \\
J_{i}^{(1)} &= (\mu_{ij}^{(1)} E_{j}^{(0)} + \lambda_{i}^{(1)} n^{(0)} - d_{ij}^{(1)} N_{j}^{(0)}) + (\mu_{ij}^{(2)} E_{j}^{(1)} + \lambda_{i}^{(2)} n^{(1)} - d_{ij}^{(2)} N_{j}^{(1)}),
\end{aligned}
\tag{27}
$$

where

$$
\begin{aligned}
c_{ijkl}^{(2)} &= \sum_{I=1}^{N} (\frac{h_I^3 - h_{I-1}^3}{3})\bar{c}_{ijkl}^{I}, \quad e_{kij}^{(2)} = \sum_{I=1}^{N} (\frac{h_I^3 - h_{I-1}^3}{3})\bar{e}_{kij}^{I}, \\
\varepsilon_{ij}^{(2)} &= \sum_{I=1}^{N} (\frac{h_I^3 - h_{I-1}^3}{3})\bar{\varepsilon}_{ij}^{I}, \quad \mu_{ij}^{(2)} = \sum_{I=1}^{N} (\frac{h_I^3 - h_{I-1}^3}{3})q^I \bar{n}^I \mu_{ij}^{I}, \\
d_{ij}^{(2)} &= \sum_{I=1}^{N} (\frac{h_I^3 - h_{I-1}^3}{3})q^I d_{ij}^{I}, \quad \lambda_{i}^{(2)} = \sum_{I=1}^{N} (\frac{h_I^3 - h_{I-1}^3}{3})q^I \mu_{ij}^{I} \bar{E}_{j}^{I}.
\end{aligned}
\tag{28}
$$

Two shear correction factors κ_1 and κ_2 can be introduced in the manner of [11] by the replacement of the following zero-order strains

$$
S_{31}^{(0)} \rightarrow \kappa_1 S_{31}^{(0),} \quad S_{32}^{(0)} \rightarrow \kappa_2 S_{32}^{(0)}.
\tag{29}
$$

The two correction factors should be determined for specific plates by requiring the two fundamental thickness-shear resonant frequencies obtained from the two-dimensional plate equations to be equal to the corresponding exact frequencies predicted by the three-dimensional equations. With shear correction factors thus determined, the two-dimensional plate equations and the exact three-dimensional equations yield the same frequencies for a particular motion, i.e., the thickness-shear vibrations of a plate in the two fundamental thickness-shear modes.

In summary, we have obtained the two-dimensional equations of motion, Gauss's law and the conservation of charge (Eq. 23), the constitutive relations (Eqs. 24 and 27) and the displacement, potential and carrier density gradients (Eqs. 17–22). With successive substitutions, Eq. (23) can be written as nine equations for the nine unknowns of $u_1^{(0)}, u_2^{(0)}, u_3^{(0)}, u_1^{(1)}, u_2^{(1)}$,

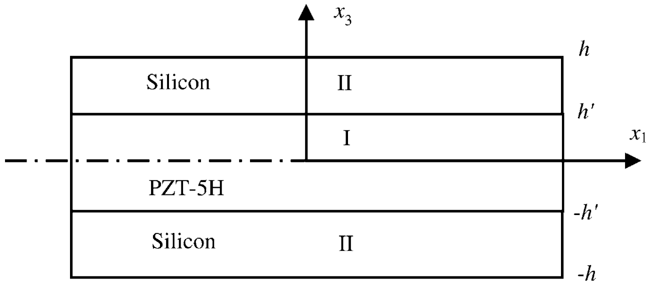

Fig. 1. A three-layered piezoelectric-semiconductor plate

$\phi^{(0)}, \phi^{(1)}, n^{(0)}$, and $n^{(1)}$. At the boundary of a plate with in-plane unit exterior normal **n** and in-plane unit tangent **s**, we may prescribe

$$\begin{aligned}
&T^{(0)}_{nn} \text{ or } u^{(0)}_n, T^{(0)}_{ns} \text{ or } u^{(0)}_s, T^{(0)}_{n3} \text{ or } u^{(0)}_3,\\
&T^{(1)}_{nn} \text{ or } u^{(1)}_n, T^{(1)}_{ns} \text{ or } u^{(1)}_s,\\
&D^{(0)}_n \text{ or } \phi^{(0)}, D^{(1)}_n \text{ or } \phi^{(1)},\\
&J^{(0)}_n \text{ or } n^{(0)}, J^{(1)}_n \text{ or } n^{(1)}.
\end{aligned} \tag{30}$$

5
Propagation of extensional waves

As an example for the application of the above equations we study the propagation of extensional waves in a symmetric composite plate as shown in Fig. 1. The ceramic layer of PZT-5H is a piezoelectric dielectric and silicon is a nonpiezoelectric semiconductor. For ceramics poled in the x_1 direction, the material tensors in Eq. (2) are special cases of the following matrices

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0\\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \begin{pmatrix} e_{11} & 0 & 0\\ e_{12} & 0 & 0\\ e_{13} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & e_{35}\\ 0 & e_{26} & 0 \end{pmatrix}^T, \begin{pmatrix} \varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{22} & 0\\ 0 & 0 & \varepsilon_{33} \end{pmatrix}, \tag{31}$$

where the superscript T indicates matrix transpose. Silicon is of m3m symmetry with

$$\begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0\\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{pmatrix}, \begin{pmatrix} \varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{11} & 0\\ 0 & 0 & \varepsilon_{11} \end{pmatrix}. \tag{32}$$

μ_{ij} and d_{ij} have the same structure as ϵ_{ij}. We consider extensional waves in the x_1 direction with $u^{(0)}_1(x_1, t), \phi^{(0)}(x_1, t)$ and $n^{(0)}(x_1, t)$. The dc bias $\bar{E}_1$ is a constant. The surfaces of the plate at $x_3 = \pm h$ are traction free and are without free charge and normal current. Then the constitutive relations take the following form

$$\begin{aligned}
T^{(0)}_{11} &= [2h'1\bar{c}^I_{11} + 2(h - h')\bar{c}^{II}_{11}]u^{(0)}_{1,1} + 2h'\bar{e}^I_{11}\phi^{(0)}_{,1},\\
D^{(0)}_1 &= 2h'\bar{e}_{11}u^{(0)}_{1,1} - [2h'\bar{\varepsilon}^I_{11} + 2(h - h')\bar{\varepsilon}^{II}_{11}]\phi^{(0)}_{,1},\\
J^{(0)}_1 &= 2(h - h')q^{II}(-\bar{n}^{II}\mu^{II}_{11}\phi^{(0)}_{,1} + \mu^{II}_{11}\bar{E}^{II}_1 n^{(0)} - d^{II}_{11}n^{(0)}_{,1}),
\end{aligned} \tag{33}$$

where

$$\begin{aligned}&\bar{c}_{11} = c_{11} - c_{13}^2/c_{33}, \quad \bar{c}_{12} = c_{12} - c_{13}c_{32}/c_{33,}\\&\bar{e}_{11} = e_{11} - e_{13}c_{31}/c_{33,} \quad \bar{e}_{12} = e_{12} - e_{13}c_{32}/c_{33,}\\&\bar{\varepsilon}_{11} = \varepsilon_{11} + e_{13}^2/c_{33,}\end{aligned} \tag{34}$$

Substituting Eq. (33) into the relevant equations of Eq. (23) yields the following equations for extension

$$[h'\bar{c}_{11}^{I} + (h-h')\bar{c}_{11}^{II}]u_{1,11}^{(0)} + h'\bar{e}_{11}^{I}\phi_{,11}^{(0)} = [h'\rho^{I} + (h-h')\rho^{II}]\ddot{u}_1^{(0)}, \tag{35}$$

electrostatics

$$h'\bar{e}_{11}^{I}u_{1,11}^{(0)} - [h'\bar{\varepsilon}_{11}^{I} + (h-h')\bar{\varepsilon}_{11}^{II}]\phi_{,11}^{(0)} = q^{II}(h-h')n^{(0)}, \tag{36}$$

and conservation of charge

$$\dot{n}^{(0)} - \bar{n}^{II}\mu_{11}^{II}\phi_{,11}^{(0)} + \mu_{11}^{II}\bar{E}_1^{II}n_{,1}^{(0)} - d_{11}^{II}n_{,11}^{(0)} = 0. \tag{37}$$

Let

$$\begin{Bmatrix} u_1^{(0)} \\ n^{(0)} \\ \phi^{(0)} \end{Bmatrix} = \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} e^{i(\xi x_1 - \omega t),} \tag{38}$$

where *A*, *B* and *C* are constants. Substitution of Eq. (38) into Eqs. (35)–(37) yields

$$\begin{bmatrix} \begin{array}{l}[h'\rho^{I} + (h-h')\rho^{II}]\omega^2 \\ -[h'\bar{c}_{11}^{I} + (h-h')\bar{c}_{11}^{II}]\xi^2\end{array} & 0 & -h'\bar{e}_{11}^{I}\xi^2 \\ -h'\bar{e}_{11}^{I}\xi^2 & -q^{II}(h-h') & [h'\bar{\varepsilon}_{11}^{I} + (h-h')\bar{\varepsilon}_{11}^{II}]\xi^2 \\ 0 & i(\mu_{11}^{II}\bar{E}_1^{II}\xi - \omega) + d_{11}^{II}\xi^2 & \bar{n}^{II}\mu_{11}^{II}\xi^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0. \tag{39}$$

For nontrivial solutions the determinant of the coefficient matrix must vanish, which leads to the following dispersion relation

$$\begin{aligned}[h'\rho^{I} + (h-h')\rho^{II}]\omega^2 &= [h'\bar{c}_{11}^{I} + (h-h')\bar{c}_{11}^{II}]\xi^2 \\ &+ \frac{(h'\bar{e}_{11}^{I}\xi^2)^2[i(\mu_{11}^{II}\bar{E}_1^{II}\xi - \omega) + d_{11}^{II}\xi^2]}{q^{II}(h-h')\bar{n}^{II}\mu_{11}^{II}\xi^2 + [i(\mu_{11}^{II}\bar{E}_1^{II}\xi - \omega) + d_{11}^{II}\xi^2][h'\bar{\varepsilon}_{11}^{I} + (h-h')\bar{\varepsilon}_{11}^{II}]\xi^2}.\end{aligned} \tag{40}$$

Equation(40) shows the wave is dispersive due to semiconduction. It also shows that ω is expected to be complex indicating a wave damped due to semiconduction. It can also be seen that wave amplification may occur when $\mu_{11}^{II}\bar{E}_1^{II}\xi - \omega$ changes it's sign or

$$\frac{\omega}{\xi} = \mu_{11}^{II}\bar{E}_1^{II}, \tag{41}$$

i.e., the acoustic wave speed is equal to the carrier drift speed [4]. When semiconduction is small, Eq. (40) can be solved approximately by an iteration procedure. As the zero-order of approximation, we neglect the small semiconduction and obtain

$$[h'\rho^{I} + (h-h')\rho^{II}]\omega_{(0)}^2 = [h'\bar{c}_{11}^{I} + (h-h')\bar{c}_{11}^{II}]\xi^2 + \frac{(h'\bar{e}_{11}^{I})^2}{h'\bar{\varepsilon}_{11}^{I} + (h-h')\bar{\varepsilon}_{11}^{II}}\xi^2, \tag{42}$$

or, in terms of the wave speed

$$v_{(0)}^2 = \frac{\omega_{(0)}^2}{\xi^2} = \frac{h'\bar{c}_{11}^I + (h-h')\bar{c}_{11}^{II}}{h'\rho^I + (h-h')\rho^{II}} + \frac{(h'\bar{e}_{11}^I)^2}{[h'\bar{\varepsilon}_{11}^I + (h-h')\bar{\varepsilon}_{11}^{II}][h'\rho^I + (h-h')\rho^{II}]}, \tag{43}$$

which is the speed of an extensional wave with the well known piezoelectrically stiffening effect. Equation (42) is nondispersive and nondissipative. For the next order we substitute Eq. (42) into the right hand side of Eq. (40) and obtain

$$\begin{aligned}[h'\rho^I + (h-h')\rho^{II}]\omega_{(1)}^2 &= [h'\bar{c}_{11}^I + (h-h')\bar{c}_{11}^{II}]\xi^2 \\ &+ \frac{(h'\bar{e}_{11}^I\xi^2)^2[i(\mu_{11}^{II}\bar{E}_1^{II}\xi - \omega_{(0)}) + d_{11}^{II}\xi^2]}{q^{II}(h-h')\bar{n}^{II}\mu_{11}^{II}\xi^2 + [i(\mu_{11}^{II}\bar{E}_1^{II}\xi - \omega_{(0)}) + d_{11}^{II}\xi^2][h'\bar{\varepsilon}_{11}^I + (h-h')\bar{\varepsilon}_{11}^{II}]\xi^2},\end{aligned} \tag{44}$$

which is dispersive and dissipative.

For PZT-5H, $\rho = 7500$ kg/m. When poling is along the x_1 axis the material matrices are [1]

$$c_{pg} = \begin{pmatrix} 11.7 & 8.41 & 8.41 & 0 & 0 & 0 \\ 8.41 & 12.6 & 7.95 & 0 & 0 & 0 \\ 8.41 & 7.95 & 12.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.325 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.3 \end{pmatrix} \times 10^{10}(\mathrm{N/m^2}), \tag{45}$$

$$[e_{ip}]^T = \begin{pmatrix} 23.3 & 0 & 0 \\ -6.5 & 0 & 0 \\ -6.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 17 \\ 0 & 17 & 0 \end{pmatrix}(\mathrm{C/m^2}), \quad \epsilon_{ij} = \begin{pmatrix} 1.302 & 0 & 0 \\ 0 & 1.505 & 0 \\ 0 & 0 & 1.505 \end{pmatrix} \times 10^{-8}(\mathrm{C/Vm}). \tag{46}$$

For silicon we have [17,18]

$$\begin{aligned}&\rho = 2332\ \mathrm{kg/m^3}, \\ &c_{11} = 16.57, \quad c_{44} = 7.956, \quad c_{12} = 6.39 \times 10^{10}\ \mathrm{N/m^2}, \\ &\varepsilon_{11} = 11.8\varepsilon_0, \quad \varepsilon_0 = 8.854 \times 10^{-12}\ \text{Farads/m}.\end{aligned} \tag{47}$$

The mobility of electrons and holes of silicon are [19]

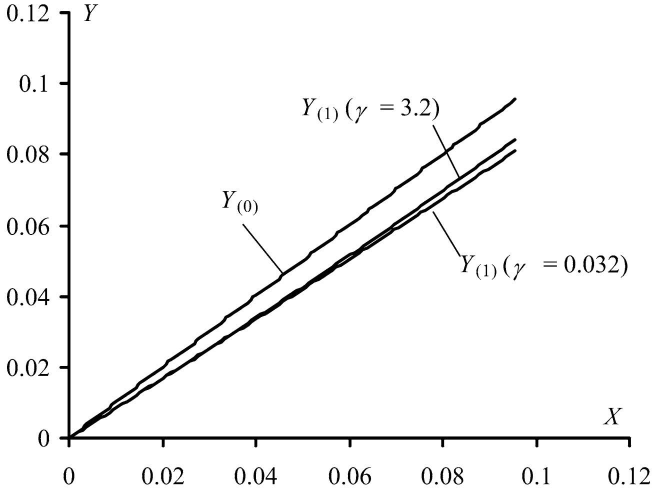

Fig. 2. The effect of semiconduction on wave speed

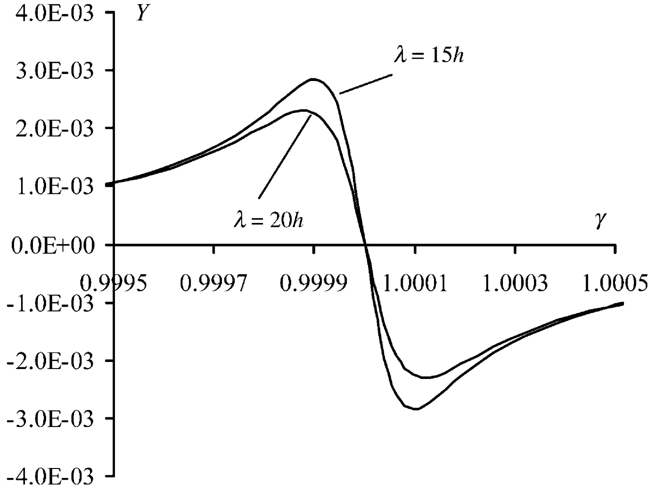

Fig. 3. The effect of semiconduction on wave attenuation

$$\mu_n = 1500, \mu_p = 480\ \text{cm}^2/\text{V-sec}\,. \tag{48}$$

μ_n for electrons will be used in our calculation. The diffusion constants can be determined from the Einstein relation [19]

$$D = \frac{kT}{q_e}\mu, \tag{49}$$

where $T(= 300\,\text{K})$ is the absolute temperature, $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, and $q_e = 1.602 \times 10^{-19}$ Coulomb is the electronic charge. We selected the thickness of the ceramic and semiconductor layers as h = 3 mm and h' = 2 mm.

We plot the real parts of $\omega_{(1)}$ and $\omega_{(2)}$ versus ξ in Fig. 2. The dimensionless wave number X and the dimensionless frequency Y of different orders are defined by

$$X = \xi\Big/\frac{\pi}{2h}, \quad Y_{(0)} = \omega_{(0)}\Big/\left(v_{(0)}\frac{\pi}{2h}\right), \quad Y_{(1)} = \text{Re}\{\omega_{(1)}\}\Big/\left(v_{(0)}\frac{\pi}{2h}\right). \tag{50}$$

γ is a dimensionless number given by

$$\gamma = \mu_{11}^{II}\bar{E}_1^{II}/v_{(0)}, \tag{51}$$

which may be considered as a normalized electric field. It also represents the ratio of the electron drift velocity and the extensional wave speed. $Y_{(0)}$ is a straight line of a nondispersive wave. $Y_{(1)}$ is in fact curved as shown by Eq. (44), representing a dispersive wave, but the dispersion is very small and cannot be seen in the figure. However, the change of wave speed (slope) caused by semiconduction can be seen clearly. This change of wave speed varies according to the dc electric biasing field. Our iteration solution is accurate in the sense that the second-order solution does not show much difference from the first-order solution, and the difference cannot be seen when plotted.

Fig. 3 shows the imaginary part of $\omega_{(1)}$ versus γ which represents $\bar{E}_1^{II}$, and $\lambda = 2\pi/\xi$ is the usual wavelength. The dimensionless number describing the decaying behavior of the waves is defined by

$$Y = \text{Im}\{\omega_{(1)}\}\Big/\left(v_{(0)}\frac{\pi}{2h}\right). \tag{52}$$

When the dc bias is large enough the decay constant becomes negative indicating wave amplification. The transition from damped waves to growing waves indeed occurs when Eq. (41) is true for $\omega_{(0)}$. Note the maximum and minimum before and after the transition; these agree qualitatively with the behavior of the one-dimensional plane waves in piezoelectric semiconductors studied in [4].

6
Conclusion

Two-dimensional equations for coupled extensional, flexural and thickness-shear motions of thin plates of laminated piezoelectric semiconductors under a dc field are obtained. It is shown that semi-conduction causes dispersion and acoustic loss in the propagation of extensional waves. The equations are useful in analyzing laminated plate structures for acoustoelectric devices.

References

1. **Auld, B.A.:** Acoustic fields and waves in solids, vol. I. Wiley New York (1973)
2. **Hutson, A.R.; White, D.L.:** Elastic wave propagation in piezoelectric semiconductors. J Appl Phys 33 (1962) 40–47
3. **Weinreich, G.; Sanders, Jr T.M.; White, H.G.:** Acoustoelectric effect in n-type Germanium. Phys Rev 114 (1959) 33–44
4. **White, D.L.:** Amplification of ultrasonic waves in piezoelectric semiconductors. J Appl Phys 33 (1962) 2547–2554
5. **Kino, G.S.:** Acoustoelectric interactions in acoustic-surface-wave devices. Proc IEEE 64 (1976) 724–748
6. **Heyman, J.S.:** Phase insensitive acoustoelectric transducer. J Acoust Soc Am 64 (1978) 243–249
7. **Busse, L.J.; Miller, J.G.:** Response characteristics of a finite aperture, phase insensitive ultrasonic receiver based upon the acoustoelectric effect. J Acoust Soc Am 70 (1981) 1370–1376
8. **Dietz, D.R.; Busse, L.J.; Fife, M.J.:** Acoustoelectric detection of ultrasound power with composite piezoelectric and semiconductor devices. IEEE Trans Ultrason Ferroelectr Freq Control 35 (1988) 146–151
9. **de Lorenzi, H.G.; Tiersten, H.F.:** On the interaction of the electromagnetic field with heat conducting deformable semiconductors. J Math Phys 16 (1975) 938–957
10. **Maugin, G.A.; Daher, N.:** Phenomenological theory of elastic semiconductors. Int J Engng Sci 24 (1986) 703–731
11. **Mindlin, R.D.:** High frequency vibrations of piezoelectric crystal plates. Int J Solids Struct 8 (1972) 895–906
12. **Lee, P.C.Y.; Syngellakis, S.; Hou, J.P.:** A two-dimensional theory for high-frequency vibrations of piezoelectric crystal plates with or without electrodes. J Appl Phys 61 (1987) 1249–1262
13. **Tiersten, H.F.:** Equations for the extension and flexure of relatively thin electrostatic plates undergoing larger electric fields. In: Lee JS, Maugin GA, Shindo Y (eds) Mechanics of electromagnetic materials and structures, AMD vol 161, MD vol. 42. ASME, New York (1993)
14. **Wang, J.; Yang, J.S.:** Higher-order theories of piezoelectric plates and applications. Appl Mech Re 53 (2000) 87–99
15. **Wauer, J.; Suherman, S.:** Thickness vibrations of a piezo-semiconducting plate layer. Int J Eng Sci 35 (1997) 1387–1404
16. **Tiersten, H.F.:** Linear piezoelectric plate vibrations. Plenum, New York (1969)
17. **Hellwege, K.H., Hellege, A.M.:** Landolt-Bornstein, Numerical data and functional relationships in science and technology. Springer Berlin Heidelberg, New York (1979)
18. **Lide, D.R.:** (eds) CRC handbook of chemistry and physics, 82nd edition. CRC Press, Cleveland, Ohio (2001–2002)
19. **Navon, D.H.:** Semiconductor microdevices and materials. CBS College Publishing, New York (1986)