Amplification of Acoustic Waves in Piezoelectric Semiconductor Shells

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ABSTRACT: Two-dimensional equations for multilayered shells of piezoelectric semiconductors are derived. The equations are used to analyze the propagation of torsional waves in a single-layered circular cylindrical shell of a piezoelectric semiconductor, and in a multilayered shell of nonconducting piezoelectrics and nonpiezoelectric semiconductors. Dispersion and dissipation due to semiconduction as well as wave amplification by a biasing DC electric field are discussed.

Key Words: piezoelectricity, semiconductor

INTRODUCTION

PIEZOELECTRIC materials are either dielectrics or semiconductors (Auld, 1973). An acoustic wave propagating in a piezoelectric crystal is usually accompanied by an electric field. When the crystal is also semiconducting, the electric field produces currents and space charge resulting in dispersion and acoustic loss (Hutson and White, 1962). The interaction between a traveling acoustic wave and mobile charges in piezoelectric semiconductors is called the acoustoelectric effect which is a special case of a more general phenomenon which may be called the wave-particle drag (Weinreich et al., 1959). It was also found that an acoustic wave traveling in a piezoelectric semiconductor can be amplified by the application of a DC electric field (White, 1962). The acoustoelectric effect can also be produced in composites of piezoelectric dielectrics and nonpiezoelectric semiconductors (Dietz et al., 1988). In these composites, the acoustoelectric effect is due to the combination of the piezoelectric effect and semiconduction in each component phase. The acoustoelectric effect and the acoustoelectric amplification of acoustic waves have led to the development of acoustoelectric devices (Heyman, 1978; Busse and Miller, 1981; Dietz et al., 1988). The basic behavior of piezoelectric semiconductors and the acoustoelectric effect can be described by a linear phenomenological theory (Hutson and White, 1962; White, 1962). More sophisticated nonlinear theories for deformable semiconductors have also been developed (de Lorenzi and Tiersten, 1975; Maugin and Daher, 1986). Due to multifield coupling and anisotropy, device modeling by these theories presents complicated mathematical problems. Exact analysis is possible only in rare cases, e.g., the one-dimensional problem of thickness vibrations of plates (Wauer and Suherman, 1997). Structural theories and numerical methods need to be developed for device modeling. Two-dimensional theories for thin piezoelectric dielectric shells have been developed (Dokmeci, 1990; Tzou, 1993; Rogacheva, 1994; Hu et al., 2002) and proved very effective in structural problems and device modeling. Single- and multilayered piezoelectric semiconductor shells can be used to produce the acoustoelectric effect for device application. In this article, we study motions of thin shells of layered piezoelectric semiconductors. The three-dimensional equations of linear piezoelectric semiconductors are summarized in the following section. Two-dimensional shell equations are then derived. Propagation of torsional waves in a single-layered piezoelectric semiconductor shell, and in a multilayered shell of piezoelectric dielectrics and nonpiezoelectric semiconductors, under a DC field, is analyzed. Finally, some conclusions are drawn.

THREE-DIMENSIONAL EQUATIONS

Consider a homogeneous, one-carrier piezoelectric semiconductor under a uniform DC electric field $\overline{E}_j$. The steady-state current, $\overline{J}_i = q\overline{n}\mu_{ij}\overline{E}_j$, where q is the carrier charge, $\overline{n}$ is the steady-state carrier density which produces electrical neutrality, and μ_{ij} is the carrier mobility. The summation convention for repeated tensor indices is used. When an acoustic wave propagates through the material, perturbations of the electric field, the carrier density, and the current are denoted

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by E_j, n, and J_i. The linear theory for small signals consists of the equations of motion, Gauss's law of electrostatics (the charge equation), and the conservation of charge (Hutson and White, 1962)

$$T_{ji,j} = \rho \ddot{u}_i, \qquad D_{i,i} = qn, \qquad q\dot{n} + J_{i,i} = 0, \tag{1}$$

where, u_i is the displacement vector, T_{ij} is the stress tensor, ρ is the mass density, and D_i is the electric displacement vector. A comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index. A superimposed dot represents differentiation with respect to time t. The above equations are accompanied by the following constitutive relations:

$$\begin{aligned} T_{ij} &= c_{ijkl}S_{kl} - e_{kij}E_k, \\ D_i &= e_{ijk}S_{jk} + \varepsilon_{ij}E_j, \\ J_i &= q\bar{n}\mu_{ij}E_j + qn\mu_{ij}\overline{E}_j - qd_{ij}N_j, \end{aligned} \tag{2}$$

where the strain tensor S_{ij}, the electric potential ϕ, and the carrier density gradient N_j are defined by:

$$S_{ij} = \frac{(u_{i,j} + u_{j,i})}{2}, \qquad E_i = -\phi_{,i}, \qquad N_j = n_{,j}. \tag{3}$$

In Equation (2), c_{ijkl}, e_{kij}, and ε_{ij} are the elastic, piezoelectric, and dielectric constants. d_{ij} are the carrier diffusion constants. With successive substitutions from Equations (2) and (3), Equation (1) can be written as five equations for $\mathbf{u}$, ϕ, and n

$$\begin{aligned} &c_{ijkl}u_{k,lj} + e_{kij}\phi_{,kj} = \rho\ddot{u}_i, \\ &e_{ikl}u_{k,li} - \varepsilon_{ij}\phi_{,ij} = qn, \\ &\dot{n} - \bar{n}\mu_{ij}\phi_{,ij} + \mu_{ij}\overline{E}_j n_{,i} - d_{ij}n_{,ij} = 0. \end{aligned} \tag{4}$$

On the boundary of a finite body with a unit outward normal n_i, the mechanical displacement u_i or the traction vector $T_{ij}n_i$, the electric potential ϕ or the normal component of the electric displacement vector D_in_i, and the carrier density n or the normal current J_in_i may be prescribed.

DERIVATION OF SHELL EQUATIONS

Consider a shell element in the reference configuration (see Figure 1). $(\alpha_1, \alpha_2, \alpha_3)$ is an orthogonal curvilinear coordinate system, α_1 and α_2 are the middle surface principal coordinates, and α_3 is the thickness coordinate. The thickness of the shell, $2h$, is much smaller than the radii of curvature R_1 and R_2 of the middle surface. The shell has N layers. The two major faces and the $N-1$ interfaces are sequentially determined by $\alpha_3 = -h = h_0, h_1, \ldots, h_{N-1}$, and $h_N = h$. A_1 and A_2 are Lame coefficients of the middle surface corresponding to α_1 and α_2.

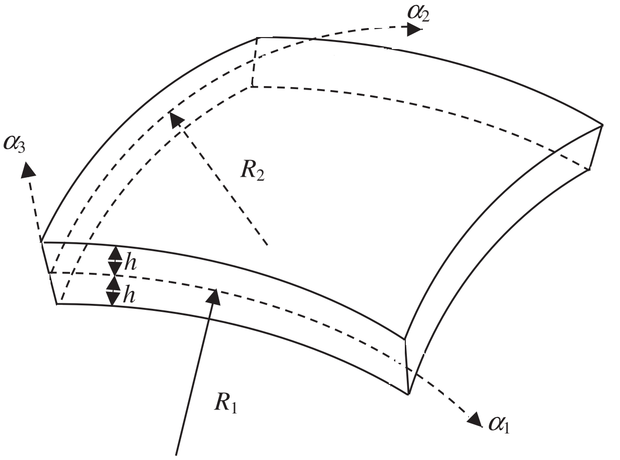

Figure 1. *A shell element and coordinate system.*

Displacement, Potential, and Carrier Density Approximation

The three-dimensional mechanical displacement, electric potential, and carrier density are approximated by

$$\begin{aligned} u_1(\alpha_1, \alpha_2, \alpha_3, t) &\cong u_1(\alpha_1, \alpha_2, t) + \beta_1(\alpha_1, \alpha_2, t)\alpha_3, \\ u_2(\alpha_1, \alpha_2, \alpha_3, t) &\cong u_2(\alpha_1, \alpha_2, t) + \beta_2(\alpha_1, \alpha_2, t)\alpha_3, \\ u_3(\alpha_1, \alpha_2, \alpha_3, t) &\cong u_3(\alpha_1, \alpha_2, t), \\ \phi(\alpha_1, \alpha_2, \alpha_3, t) &\cong \phi^{(0)}(\alpha_1, \alpha_2, t) + \alpha_3\phi^{(1)}(t), \\ n(\alpha_1, \alpha_2, \alpha_3, t) &\cong n^{(0)}(\alpha_1, \alpha_2, t). \end{aligned} \tag{5}$$

When the shell is electroded, $\phi^{(0)} = 0$, and $\phi^{(1)} = V(t)$ is the voltage across the shell thickness. When the shell is unelectroded, to the lowest order $\phi = \phi^{(0)}$, and $\phi^{(1)} = 0$. Note that we have used the same u_i for both the three-dimensional displacements and middle surface displacements, which should not cause any confusion. For a classical thin shell without transverse shear strains ($S_{13} = S_{23} = 0$), the rotations β_1 and β_2 can be expressed in terms of the middle surface displacements as (Tzou, 1993)

$$\beta_1 = -\frac{\partial u_3}{A_1\partial\alpha_1} + \frac{u_1}{R_1}, \qquad \beta_2 = -\frac{\partial u_3}{A_2\partial\alpha_2} + \frac{u_2}{R_2}. \tag{6}$$

Gradient Equations

The in-plane strain components S_{ab} $(a, b = 1, 2)$ have the following expression

$$S_{ab} = S_{ab}^{(0)} + \alpha_3\kappa_{ab}, \tag{7}$$

where, the middle surface strain components $S_{ab}^{(0)}$ and curvatures κ_{ab} are given by (Tzou, 1993)

$$\begin{aligned} S_{11}^{(0)} &= \frac{1}{A_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{u_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} + \kappa_{11} u_3, \\ S_{22}^{(0)} &= \frac{1}{A_2}\frac{\partial u_2}{\partial \alpha_2} + \frac{u_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} + \kappa_{22} u_3, \\ 2S_{12}^{(0)} &= \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left(\frac{u_2}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left(\frac{u_1}{A_1}\right), \end{aligned} \tag{8}$$

and

$$\begin{aligned} \kappa_{11} &= \frac{1}{A_1}\frac{\partial \beta_1}{\partial \alpha_1} + \frac{\beta_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}, \\ \kappa_{22} &= \frac{1}{A_2}\frac{\partial \beta_2}{\partial \alpha_2} + \frac{\beta_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}, \\ \kappa_{12} &= \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left(\frac{\beta_2}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left(\frac{\beta_1}{A_1}\right). \end{aligned} \tag{9}$$

The electric field and carrier density gradients are

$$\begin{aligned} &E_1^{(0)} = -\frac{1}{A_1}\frac{\partial}{\partial \alpha_1}(\phi^{(0)}), \qquad E_2^{(0)} = -\frac{1}{A_2}\frac{\partial}{\partial \alpha_2}(\phi^{(0)}), \\ &E_3^{(0)} = -\phi^{(1)} \end{aligned} \tag{10}$$

and

$$N_1^{(0)} = \frac{1}{A_1}\frac{\partial}{\partial \alpha_1}(n^{(0)}), \qquad N_2^{(0)} = \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}(n^{(0)}), \qquad N_3^{(0)} = 0. \tag{11}$$

Divergence Equations

The shell equations of motion (Tzou, 1993), Gauss's law (Tzou, 1993), and the conservation of charge take the following form:

$$\begin{aligned} &\frac{\partial(N_{11}A_2)}{\partial \alpha_1} + \frac{\partial(N_{21}A_1)}{\partial \alpha_2} + N_{12}\frac{\partial A_1}{\partial \alpha_2} - N_{22}\frac{\partial A_2}{\partial \alpha_1} + Q_{13}A_1A_2\frac{1}{R_1} \\ &\quad + F_1^{(0)} = \rho^{(0)}A_1A_2\ddot{u}_1, \\ &\frac{\partial(N_{12}A_2)}{\partial \alpha_1} + \frac{\partial(N_{22}A_1)}{\partial \alpha_2} + N_{21}\frac{\partial A_2}{\partial \alpha_1} - N_{11}\frac{\partial A_1}{\partial \alpha_2} + Q_{23}A_1A_2\frac{1}{R_2} \\ &\quad + F_2^{(0)} = \rho^{(0)}A_1A_2\ddot{u}_2, \\ &\frac{\partial}{\partial \alpha_1}(Q_{13}A_2) + \frac{\partial}{\partial \alpha_2}(Q_{23}A_1) - N_{11}\frac{A_1A_2}{R_1} - N_{22}\frac{A_1A_2}{R_2} \\ &\quad + F_3^{(0)} = \rho^{(0)}A_1A_2\ddot{u}_3, \\ &\frac{\partial(M_{11}A_2)}{\partial \alpha_1} + \frac{\partial(M_{21}A_1)}{\partial \alpha_2} + M_{12}\frac{\partial A_1}{\partial \alpha_2} - M_{22}\frac{\partial A_2}{\partial \alpha_1} \\ &\quad - Q_{13}A_1A_2 = 0, \\ &\frac{\partial(M_{12}A_2)}{\partial \alpha_1} + \frac{\partial(M_{22}A_1)}{\partial \alpha_2} + M_{21}\frac{\partial A_2}{\partial \alpha_1} - M_{11}\frac{\partial A_1}{\partial \alpha_2} \\ &\quad - Q_{23}A_1A_2 = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &\frac{\partial(D_1^{(0)}A_2)}{\partial \alpha_1} + \frac{\partial(D_2^{(0)}A_1)}{\partial \alpha_2} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)A_1A_2D_3^{(0)} + D^{(0)} \\ &\quad = q^{(0)}A_1A_2n^{(0)}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} &q^{(0)}A_1A_2\dot{n}^{(0)} + \frac{\partial(J_1^{(0)}A_2)}{\partial \alpha_1} + \frac{\partial(J_2^{(0)}A_1)}{\partial \alpha_2} \\ &\quad + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)A_1A_2J_3^{(0)} + J^{(0)} = 0, \end{aligned} \tag{14}$$

where the mass and charge per unit area of the middle surface are given by

$$\begin{aligned} \rho^{(0)} &= \int_{-h}^{h} \rho\, d\alpha_3 = \sum_{I=1}^{N} \rho^I (h_I - h_{I-1}), \\ q^{(0)} &= \int_{-h}^{h} q\, d\alpha_3 = \sum_{I=1}^{N} q^I (h_I - h_{I-1}). \end{aligned} \tag{15}$$

The resultants and loads are defined by

$$\begin{aligned} \{N_{ab}, Q_{a3}, D_a^{(0)}, J_a^{(0)}\} &= \int_{-h}^{h} \{T_{ab}, T_{a3}, D_a, J_a\} d\alpha_3, \\ M_{ab} &= \int_{-h}^{h} T_{ab}\alpha_3\, d\alpha_3, \\ \{F_j^{(0)}, D^{(0)}, J^{(0)}\} &= \left[A_1A_2\{T_{3j}, D_3, J_3\}\right]_{-h}^{h}. \end{aligned} \tag{16}$$

Constitutive Relations

Since the shell is assumed to be thin, we make the stress relaxation approximation of vanishing normal stress $T_{33} = 0$. This implies, through Equation $(2)_1$ by setting $i = j = 3$, the following expression for $u_{3,3}$ in terms of other components of the displacement and potential gradients

$$S_{33} = -\frac{1}{c_{3333}}(c_{33kl}S_{kl} - c_{3333}S_{33} - e_{k33}E_k). \tag{17}$$

Stress relaxation for thin anisotropic or piezoelectric plates or shells can be made in ways more sophisticated than the one already described, also involving T_{31} and T_{32} (Mindlin, 1972). That is not our main interest here. The relaxation involving T_{33} is the major relaxation because in anisotropic plates, couplings among extensions in different directions are much stronger than couplings between extensions and shears. In Equation (17), S_{33} has been eliminated on the right-hand side because when $i = j = 3$, the two terms containing S_{33} cancel with each other. From Equation (17), the thickness expansion or contraction accompanying the extension and flexure of the shell due to the Poisson's effect can be found if needed. Substituting Equation (17) back into Equations $(2)_{1,2}$, we obtain the following constitutive relations relaxed for thin plates

$$
\begin{aligned}
T_{ij} &= \bar{c}_{ijkl}S_{kl} - \bar{e}_{kij}E_k, \\
D_i &= \bar{e}_{ikl}S_{kl} + \bar{\varepsilon}_{ij}E_j,
\end{aligned}
\tag{18}
$$

where the relaxed material constants are defined by

$$
\begin{aligned}
\bar{c}_{ijkl} &= c_{ijkl} - \frac{c_{ij33}c_{33kl}}{c_{3333}}, \\
\bar{e}_{kij} &= e_{kij} - \frac{e_{k33}c_{33ij}}{c_{3333}}, \\
\bar{\varepsilon}_{ij} &= \varepsilon_{ij} + \frac{e_{i33}e_{j33}}{c_{3333}}.
\end{aligned}
\tag{19}
$$

We note that the right-hand side of Equation (18) does not contain S_{33}, and $T_{33} = 0$ is automatically satisfied by Equation (18). Furthermore, since for thin shells the transverse shear strains S_{13} and S_{23} are taken to be zero, Equation (18) can be written as:

$$
\begin{aligned}
T_{ij} &= \bar{c}_{ijab}S_{ab} - \bar{e}_{kij}E_k, \\
D_i &= \bar{e}_{iab}S_{ab} + \bar{\varepsilon}_{ij}E_j, \qquad a,b = 1,2.
\end{aligned}
\tag{20}
$$

Substitution of Equation (20) into Equations $(16)_{1,2}$ yields the following shell constitutive relations:

$$
\begin{aligned}
N_{ab} &= \int_{-h}^{h} T_{ab}\, d\alpha_3 = c^{(0)}_{abcd}S^{(0)}_{cd} + c^{(1)}_{abcd}\kappa_{cd} - e^{(0)}_{kab}E^{(0)}_k, \\
M_{ab} &= \int_{-h}^{h} T_{ab}\alpha_3\, d\alpha_3 = c^{(1)}_{abcd}S^{(0)}_{cd} + c^{(2)}_{abcd}\kappa_{cd} - e^{(1)}_{kab}E^{(0)}_k, \\
D^{(0)}_a &= \int_{-h}^{h} D_a\, d\alpha_3 = e^{(0)}_{acd}S^{(0)}_{cd} + e^{(1)}_{acd}\kappa_{cd} + \varepsilon^{(0)}_{ak}E^{(0)}_k, \\
J^{(0)}_a &= \int_{-h}^{h} J_a\, d\alpha_3 = \mu^{(0)}_{ak}E^{(0)}_k + \mu^{(0)}_a n^{(0)} - d^{(0)}_{ab}N^{(0)}_b,
\end{aligned}
\tag{21}
$$

where

$$
\begin{aligned}
c^{(0)}_{abcd} &= \int_{-h}^{h} \bar{c}_{abcd}\, d\alpha_3 = \sum_{I=1}^{N} \bar{c}^I_{abcd}(h_I - h_{I-1}), \\
c^{(1)}_{abcd} &= \int_{-h}^{h} \bar{c}_{abcd}\alpha_3\, d\alpha_3 = \sum_{I=1}^{N} \bar{c}^I_{abcd}\frac{h_I^2 - h_{I-1}^2}{2}, \\
c^{(2)}_{abcd} &= \int_{-h}^{h} \bar{c}_{abcd}\alpha_3^2\, d\alpha = \sum_{I=1}^{N} \bar{c}^I_{abcd}\frac{h_I^3 - h_{I-1}^3}{3},
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
e^{(0)}_{kab} &= \int_{-h}^{h} \bar{e}_{kab}\, d\alpha_3 = \sum_{I=1}^{N} \bar{e}^I_{kab}(h_I - h_{I-1}), \\
e^{(1)}_{kab} &= \int_{-h}^{h} \bar{e}_{kab}\alpha_3\, d\alpha = \sum_{I=1}^{N} \bar{e}^I_{kab}\frac{h_I^2 - h_{I-1}^2}{2}, \\
\varepsilon^{(0)}_{ak} &= \int_{-h}^{h} \bar{\varepsilon}_{ak}\, d\alpha_3 = \sum_{I=1}^{N} \bar{\varepsilon}^I_{ak}(h_I - h_{I-1}),
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\mu^{(0)}_{ak} &= \int_{-h}^{h} q\bar{n}\mu_{ak}\, d\alpha_3 = \sum_{I=1}^{N} q^I\bar{n}^I\mu^I_{ak}(h_I - h_{I-1}), \\
\mu^{(0)}_a &= \int_{-h}^{h} q\mu_{aj}\bar{E}_j d\alpha_3 = \sum_{I=1}^{N} q^I\mu^I_{aj}\bar{E}^I_j(h_I - h_{I-1}), \\
d^{(0)}_{ab} &= \int_{-h}^{h} q d_{ab}\, d\alpha_3 = \sum_{I=1}^{N} q^I d^I_{ab}(h_I - h_{I-1}).
\end{aligned}
\tag{24}
$$

The transverse shear resultants Q_{a3} are determined from Equations $(12)_{4,5}$. In summary, we have obtained field Equations (12)–(14), gradient relations Equations (8)–(11), and constitutive relations Equation (21). With successive substitutions from Equation (21), Equations $(12)_{4,5}$ and Equations (8)–(11), we can write Equations $(12)_{1,2,3}$, (13), and (14) as five equations for u_i, $\phi^{(0)}$, and $n^{(0)}$. Boundary conditions include the usual ones for a classical elastic shell (Tzou, 1993), $\phi^{(0)}$ or normal $\mathbf{D}^{(0)}$, and $n^{(0)}$ or normal $\mathbf{J}^{(0)}$.

TORSIONAL WAVES IN CIRCULAR CYLINDRICAL SHELLS

Cylindrical shells are common shapes for electromechanical transducers. As applications of the equations obtained, we study the propagation of torsional waves in circular cylindrical shells. Consider an unelectroded circular cylindrical shell with middle surface radius R and thickness $2h$. The cylindrical coordinates (r, θ, z) correspond to $(3, 1, 2)$ (see Figure 2). The Lame coefficients of the middle surface are $A_1 = R$ and $A_2 = 1$. The radii of curvature of middle surface are $R_1 = R$ and $R_2 = \infty$. For torsional waves, the relevant fields are $u_1(\alpha_2, t)$, $\phi^{(0)}(\alpha_2, t)$, and $n^{(0)}(\alpha_2, t)$. Equation (6) simplifies to

$$
\beta_1 = \frac{u_1}{R}, \qquad \beta_2 = 0. \tag{25}
$$

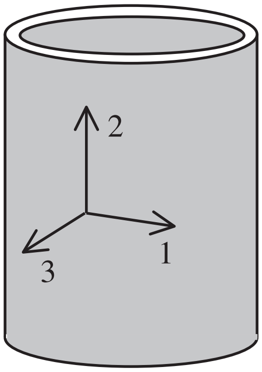

Figure 2. A circular cylindrical shell and coordinate system.

κ_{ab} and $S_{ab}^{(0)}$ take the following form:

$$\kappa_{11} = 0, \qquad \kappa_{22} = 0, \qquad \kappa_{12} = \frac{1}{R}\frac{\partial u_1}{\partial \alpha_2}, \tag{26}$$

$$S_{11}^{(0)} = 0, \qquad S_{22}^{(0)} = 0, \qquad S_{12}^{(0)} = \frac{1}{2}\frac{\partial u_1}{\partial \alpha_2}. \tag{27}$$

The electric field and carrier density gradient are

$$E_1^{(0)} = 0, \qquad E_2^{(0)} = -\frac{\partial \phi^{(0)}}{\partial \alpha_2}, \qquad E_3^{(0)} = 0, \tag{28}$$

$$N_1^{(0)} = 0, \qquad N_2^{(0)} = \frac{\partial n^{(0)}}{\partial \alpha_2}, \qquad N_3^{(0)} = 0. \tag{29}$$

We consider the case when the cylindrical surfaces are without any mechanical and electrical loads, hence

$$\{F_j^{(0)},\ D^{(0)},\ J^{(0)}\} = 0. \tag{30}$$

The biasing DC electric field is with only one component $\overline{E}_2$. We consider two examples of different materials. The material constants of the materials to be considered are special cases of the following matrices

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \quad \begin{pmatrix} e_{11} & 0 & 0 \\ e_{12} & 0 & 0 \\ e_{13} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e_{35} \\ 0 & e_{26} & 0 \end{pmatrix}^T, \quad \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}, \tag{31}$$

where the superscript T indicates matrix transpose. μ_{ij} and d_{ij} have the same structure as ε_{ij}. For a single-layered shell or a multilayered shell symmetrically laminated about its middle surface, the relevant equation of motion (Equation $(12)_1$), Gauss's law (Equation (13)), and the conservation of charge (Equation (14)) take the following form:

$$\begin{aligned} &\sum_{I=1}^{N} c_{66}^I (h_I - h_{I-1}) \frac{\partial^2 u_1}{\partial \alpha_2^2} + \sum_{I=1}^{N} e_{26}^I (h_I - h_{I-1}) \frac{\partial^2 \phi^{(0)}}{\partial \alpha_2^2} \\ &= \sum_{I=1}^{N} \rho^I (h_I - h_{I-1}) \ddot{u}_1, \qquad \sum_{I=1}^{N} e_{26}^I (h_I - h_{I-1}) \frac{\partial^2 u_1}{\partial \alpha_2^2} \\ &\quad - \sum_{I=1}^{N} \varepsilon_{22}^I (h_I - h_{I-1}) \frac{\partial^2 \phi^{(0)}}{\partial \alpha_2^2} = \sum_{I=1}^{N} q^I (h_I - h_{I-1}) n^{(0)}, \\ &\sum_{I=1}^{N} q^I (h_I - h_{I-1}) \overline{n}^I \mu_{22}^I \frac{\partial^2 \phi^{(0)}}{\partial \alpha_2^2} - \sum_{I=1}^{N} q^I (h_I - h_{I-1}) \overline{E}_2^I \mu_{22}^I \\ &\quad \times \frac{\partial n^{(0)}}{\partial \alpha_2} + \sum_{I=1}^{N} q^I \left[(h_I - h_{I-1}) d_{22}^I \left(\frac{\partial^2 n^{(0)}}{\partial \alpha_2^2} \right) \right] \\ &= \sum_{I=1}^{N} q^I (h_I - h_{I-1}) \dot{n}^{(0)}. \end{aligned} \tag{32}$$

Let

$$\begin{Bmatrix} u_1^{(0)} \\ n^{(0)} \\ \phi^{(0)} \end{Bmatrix} = \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} e^{i(\xi \alpha_2 - \omega t)}, \tag{33}$$

where A, B, and C are constants. Substitution of Equation (33) into Equation (32) yields

$$\begin{bmatrix} \sum\limits_{I=1}^{N} c_{66}^I (h_I - h_{I-1}) \xi^2 - \sum\limits_{I=1}^{N} \rho^I (h_I - h_{I-1}) \omega^2 & 0 & \sum\limits_{I=1}^{N} e_{26}^I (h_I - h_{I-1}) \xi^2 \\ -\sum\limits_{I=1}^{N} e_{26}^I (h_I - h_{I-1}) \xi^2 & -\sum\limits_{I=1}^{N} q^I (h_I - h_{I-1}) & \sum\limits_{I=1}^{N} \varepsilon_{22}^I (h_I - h_{I-1}) \xi^2 \\ 0 & \sum\limits_{I=1}^{N} q^I (h_I - h_{I-1}) (d_{22}^I \xi^2 + \overline{E}_2^I \mu_{22}^I i\xi - i\omega) & \sum\limits_{I=1}^{N} q^I (h_I - h_{I-1}) \overline{n}^I \mu_{22}^I \xi^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0 \tag{34}$$

For nontrivial solutions, the determinant of the coefficient matrix has to vanish, which gives the following dispersion relation:

$$\begin{aligned} &\sum_{I=1}^{N} \rho^I (h_I - h_{I-1}) \omega^2 = \sum_{I=1}^{N} c_{66}^I (h_I - h_{I-1}) \xi^2 \\ &\quad + \left[\sum_{I=1}^{N} e_{26}^I (h_I - h_{I-1}) \right]^2 \\ &\quad \times \left[\sum_{I=1}^{N} q^I (h_I - h_{I-1}) (d_{22}^I \xi^2 + \overline{E}_2^I \mu_{22}^I i\xi - i\omega) \right] \xi^2 \Big/ \\ &\quad \left\{ \left[\sum_{I=1}^{N} q^I (h_I - h_{I-1}) \right] \left[\sum_{I=1}^{N} q^I (h_I - h_{I-1}) \overline{n}^I \mu_{22}^I \right] \right. \\ &\quad + \left[\sum_{I=1}^{N} \varepsilon_{22}^I (h_I - h_{I-1}) \right] \\ &\quad \left. \times \left[\sum_{I=1}^{N} q^I (h_I - h_{I-1}) (d_{22}^I \xi^2 + \overline{E}_2^I \mu_{22}^I i\xi - i\omega) \right] \right\}. \end{aligned} \tag{35}$$

A Single-layered Shell

Quite a few piezoelectric semiconductors are of 6 mm symmetry. This includes, for example, widely used Beryllium Oxide (BeO), Cadmium Selenide (CdSe), Cadmium Sulfide (CdS), Zinc Oxide (ZnO), and Zinc Sulfide (ZnS) (Auld, 1973). For crystals of 6 mm symmetry, when the sixfold axis is along the x_1 axis, Equation (35) reduces to

$$\rho\omega^2 = c_{66}\xi^2 + \frac{e_{26}^2[d_{22}\xi^2 + i(\overline{E}_2\mu_{22}\xi - \omega)]\xi^2}{q\overline{n}\mu_{22} + \varepsilon_{22}[d_{22}\xi^2 + i(\overline{E}_2\mu_{22}\xi - \omega)]}. \quad (36)$$

Equation (36) shows that the wave is dispersive due to semiconduction. It also shows that ω is complex, indicating a wave damped due to semiconduction. Wave amplification may occur when the imaginary part of ω changes it sign, or when $\mu_{22}\overline{E}_2\xi - \omega$ changes its sign, i.e.,

$$\frac{\omega}{\xi} = \mu_{22}\overline{E}_2. \quad (37)$$

Equation (37) shows that wave amplification may occur when the acoustic wave speed is equal to the carrier drift speed. When semiconduction is small, Equation (36) can be solved approximately by an iteration procedure. As the zero-order approximation, we neglect the small semiconduction and obtain

$$\rho\omega_{(0)}^2 = \hat{c}_{66}\xi^2, \qquad \hat{c}_{66} = c_{66} + \frac{e_{26}^2}{\varepsilon_{22}}. \quad (38)$$

Or, in terms of the wave speed

$$v_{(0)}^2 = \frac{\omega_{(0)}^2}{\xi^2} = \frac{\hat{c}_{66}}{\rho}, \quad (39)$$

which is the speed of a torsional wave with the well-known piezoelectric stiffening effect. Equation (38) is nondispersive and nondissipative. For the next order or approximation, we substitute Equation (38) into the right-hand side of Equation (36) and obtain

$$\rho\omega_{(1)}^2 = c_{66}\xi^2 + \frac{e_{26}^2[d_{22}\xi^2 + i(\overline{E}_2\mu_{22}\xi - \omega_{(0)})]\xi^2}{q\overline{n}\mu_{22} + \varepsilon_{22}[d_{22}\xi^2 + i(\overline{E}_2\mu_{22}\xi - \omega_{(0)})]}, \quad (40)$$

which is dispersive and dissipative.

For numerical results, we consider CdS with (Auld, 1973; Gualtieri et al., 1994)

$$\begin{aligned}
&\rho = 4820\ \mathrm{kg/m^3},\\
&c_{11} = 9.38, \quad c_{22} = 9.07, \quad c_{66} = 1.504,\\
&c_{12} = 5.10, \quad c_{13} = 5.81 \times 10^{10}\ \mathrm{N/m^2},\\
&e_{11} = -0.44, \quad e_{12} = -0.24, \quad e_{26} = -0.21\ \mathrm{C/m^2},\\
&\varepsilon_{11} = 9.53\varepsilon_0, \quad \varepsilon_{22} = 9.02\varepsilon_0, \quad \varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}.
\end{aligned} \quad (41)$$

The mobility of electrons and holes in CdS at 300 K are (Navon, 1986)

$$\mu_n = 340, \qquad \mu_p = 50\ \mathrm{cm^2/V\ s}. \quad (42)$$

We consider electrons with μ_n. The diffusion constants can be determined from the Einstein relation (Navon, 1986)

$$D = \frac{kT}{q_e}\mu, \quad (43)$$

where, T is the absolute temperature, and k the Boltzmann constant. At room temperature, $kT/q_e = 0.026\ \mathrm{V}$ (Navon, 1986). $q_e = 1.602 \times 10^{-19}\ \mathrm{C}$ is the electronic charge. For geometric parameters, we choose $R = 10$ mm and $2h = 1$ mm.

We plot the real parts of $\omega_{(0)}$ and $\omega_{(1)}$ versus ξ in Figure 3. The dimensionless wave number X and the dimensionless frequency Y of different orders are defined by:

$$X = \frac{\xi}{\pi/2h}, \qquad Y_{(0)} = \frac{\omega_{(0)}}{\left(v_{(0)}\pi/2h\right)}, \qquad Y_{(1)} = \frac{\mathrm{Re}\{\omega_{(1)}\}}{\left(v_{(0)}\pi/2h\right)}. \quad (44)$$

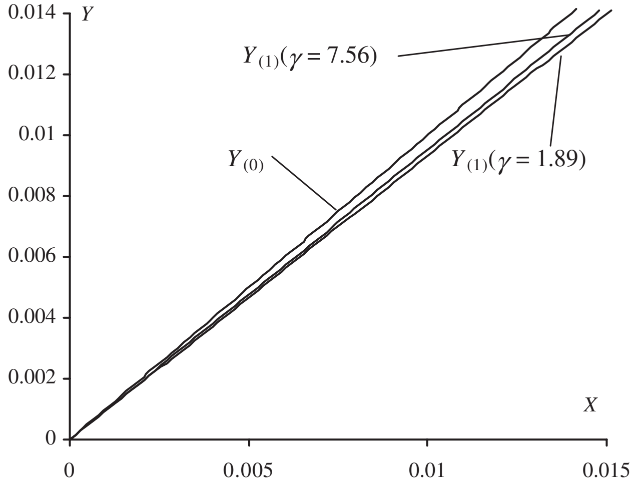

Figure 3. *Effect of semiconduction on wave speed (CdS).*

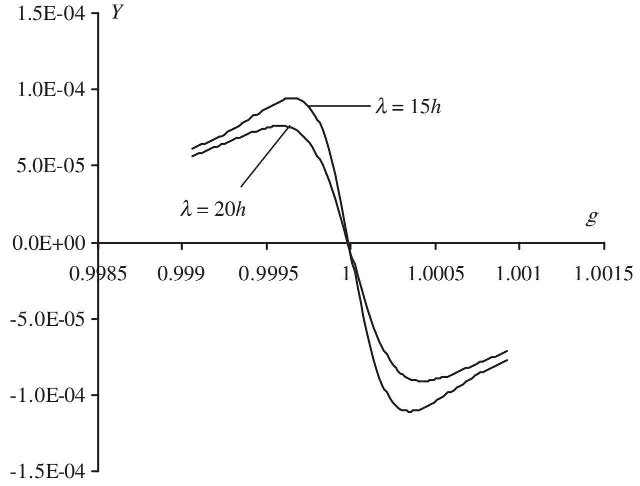

Figure 4. *Effect of semiconduction on wave attenuation (CdS).*

γ is a dimensionless number given by:

$$\gamma = \frac{\mu_{22}\overline{E}_2}{v_{(0)}}, \tag{45}$$

which may be considered as a normalized electric field. γ represents the ratio of the electron drift velocity and the torsional wave speed. $Y_{(0)}$ is a straight line of a nondispersive wave. $Y_{(1)}$ is in fact curved as shown in Equation (40), which represents a dispersive wave. Although the curvature of $Y_{(1)}$ cannot be seen in the figure, the change of slopes can be seen clearly, which represents change of wave speed. The figure also shows that the change of wave speed due to semiconduction varies according to the DC electric biasing field. Our iteration solution is accurate in the sense that the second-order solution does not show much difference from the first-order solution. The difference cannot be seen if plotted in Figure 3.

Figure 4 shows the imaginary part of $\omega_{(1)}$ versus γ. $\lambda = 2\pi/\xi$ is the usual wavelength. The dimensionless number describing the decaying behavior of the waves is defined by

$$Y = \frac{\mathrm{Im}\{\omega_{(1)}\}}{\left(v_{(0)}\pi/2h\right)}. \tag{46}$$

When the DC bias is large enough, the decay constant becomes negative, indicating wave amplification. The transition from damped waves to growing waves indeed occurs when Equation (37) is true for $\omega_{(0)}$.

A Laminated Shell

Next consider the propagation of torsional waves in a three-layered composite shell of a ceramic layer between two silicon layers. The ceramic layer of PZT-5H is a piezoelectric dielectric. Silicon is of cubic (m^{3}m) symmetry and is a nonpiezoelectric semiconductor. Then Equation (35) reduces to

$$\begin{aligned}
[h_1\rho^I + (h-h_1)\rho^{II}]\omega^2 &= [h_1 c^I_{66} + (h-h_1)c^{II}_{66}]\xi^2 \\
&+ \Big[(h_1 e^I_{26})^2\big[d^{II}_{22}\xi^2 + i(\overline{E}^{II}_2\mu^{II}_{22}\xi - \omega)\big]\xi^2\Big]\Big/ \\
&\Big[q^{II}(h-h_1)\overline{n}^{II}\mu^{II}_{22} + \big[h_1\varepsilon^I_{22} + (h-h_1)\varepsilon^{II}_{22}\big] \\
&\times\big[d^{II}_{22}\xi^2 + i(\overline{E}^{II}_2\mu^{II}_{22}\xi - \omega)\big]\Big],
\end{aligned} \tag{47}$$

where superscript I is for PZT-5H and II is for silicon. Wave amplification may occur when $\mu^{II}_{22}\overline{E}^{II}_2\xi - \omega$ changes its sign, or

$$\frac{\omega}{\xi} = \mu^{\mathrm{II}}_{22}\overline{E}^{\mathrm{II}}_2. \tag{48}$$

The zero-order of approximation gives

$$\begin{aligned}
[h_1\rho^I + (h-h_1)\rho^{II}]\omega^2_{(0)} &= [h_1 c^I_{66} + (h-h_1)c^{II}_{66}]\xi^2 \\
&+ \frac{(h_1 e^I_{26})^2\xi^2}{h_1\varepsilon^I_{22} + (h-h_1)\varepsilon^{II}_{22}}
\end{aligned} \tag{49}$$

In terms of wave speed, Equation (49) becomes

$$\begin{aligned}
v^2_{(0)} = \frac{\omega^2_{(0)}}{\xi^2} &= \frac{h_1 c^I_{66} + (h-h_1)c^{II}_{66}}{h_1\rho^I + (h-h_1)\rho^{II}} \\
&+ \frac{(h_1 e^I_{26})^2}{[h_1\rho^I + (h-h_1)\rho^{II}][h_1\varepsilon^I_{22} + (h-h_1)\varepsilon^{II}_{22}]}.
\end{aligned} \tag{50}$$

For the next order of approximation, we substitute Equation (49) into the right-hand side of Equation (47) and obtain

$$\begin{aligned}
[h_1\rho^I + (h-h_1)\rho^{II}]\omega^2_{(1)} &= [h_1 c^I_{66} + (h-h_1)c^{II}_{66}]\xi^2 \\
&+ \Big[(h_1 e^I_{26})^2\big[d^{II}_{22}\xi^2 + i(\overline{E}^{II}_2\mu^{II}_{22}\xi - \omega_{(0)})\big]\xi^2\Big]\Big/ \\
&\Big[q^{II}(h-h_1)\overline{n}^{II}\mu^{II}_{22} + \big[h_1\varepsilon^I_{22} + (h-h_1)\varepsilon^{II}_{22}\big] \\
&\times\big[d^{II}_{22}\xi^2 + i(\overline{E}^{II}_2\mu^{II}_{22}\xi - \omega_{(0)})\big]\Big],
\end{aligned} \tag{51}$$

which is dispersive and dissipative.

For PZT-5H $\rho = 7500\,\mathrm{kg/m^3}$. When poling is along the x_1-axis, the material matrices are (Auld, 1973)

$$c_{pq} = \begin{pmatrix}
11.7 & 8.41 & 8.41 & 0 & 0 & 0 \\
8.41 & 12.6 & 7.95 & 0 & 0 & 0 \\
8.41 & 7.95 & 12.6 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.325 & 0 & 0 \\
0 & 0 & 0 & 0 & 2.3 & 0 \\
0 & 0 & 0 & 0 & 0 & 2.3
\end{pmatrix} \times 10^{10}\ (\mathrm{N/m^2}), \tag{52}$$

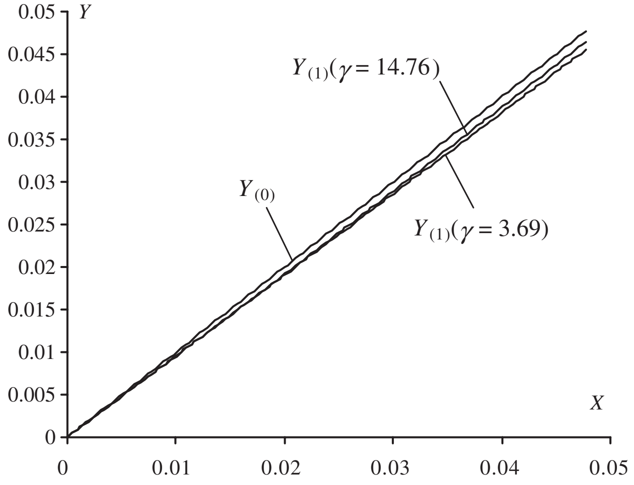

Figure 5. *Effect of semiconduction on wave speed (PZT-5H and Si).*

$$[e_{ip}]^T = \begin{pmatrix} 23.3 & 0 & 0 \\ -6.5 & 0 & 0 \\ -6.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 17 \\ 0 & 17 & 0 \end{pmatrix} (\mathrm{C/m^2}),$$

$$\varepsilon_{ij} = \begin{pmatrix} 1.302 & 0 & 0 \\ 0 & 1.505 & 0 \\ 0 & 0 & 1.505 \end{pmatrix} \times 10^{-8}\ (\mathrm{C/V\,m}). \tag{53}$$

For silicon we have (Hellwege and Hellege, 1979; Lide, 2001–2002)

$$\rho = 2332\ \mathrm{kg/m^3},$$
$$c_{11} = 16.57, \quad c_{44} = 7.956, \quad c_{12} = 6.39 \times 10^{10}\ \mathrm{N/m^2},$$
$$\varepsilon_{11} = 11.8\varepsilon_0, \quad \varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}. \tag{54}$$

The mobility of electrons and holes in silicon are (Navon, 1986)

$$\mu_n = 1500, \qquad \mu_p = 480\ \mathrm{cm^2/V\,s}. \tag{55}$$

μ_n for electrons is used in our calculation. For geometric parameters we choose $R = 10$ mm, and that the ceramic and silicon layers all have the same thickness of 1 mm.

We plot the real parts of $\omega_{(1)}$ and $\omega_{(2)}$ versus ξ in Figure 5. γ is now given by:

$$\gamma = \frac{\mu_{22}^{II}\overline{E}_2^{II}}{v_{(0)}}. \tag{56}$$

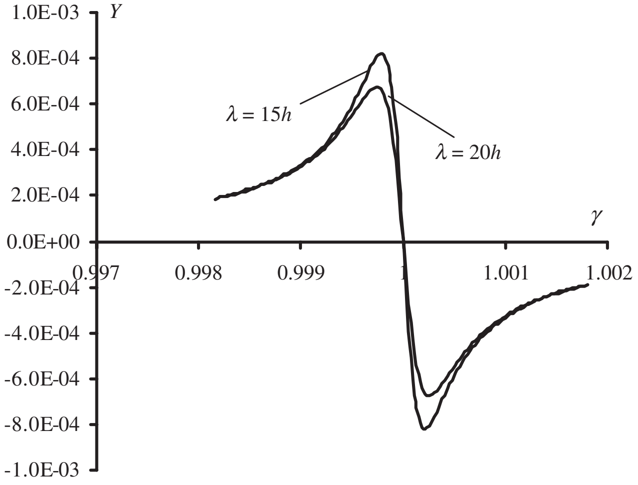

Figure 6. *Effect of semiconduction on wave attenuation (PZT-5H and Si).*

Figure 6 shows the imaginary part of $\omega_{(1)}$ versus γ. Figures 5 and 6 are qualitatively the same as Figures 3 and 4.

CONCLUSIONS

Two-dimensional equations for thin shells of laminated piezoelectric semiconductors under a DC electric field are obtained. Interaction of acoustic wave and semiconduction can occur in piezoelectric semiconductors and composites of nonconducting piezoelectrics and nonpiezoelectric semiconductors. It is shown that semiconduction causes dispersion and acoustic loss in the propagation of torsional waves, and these waves can be amplified by a DC field. The equations are useful in analyzing piezoelectric semiconductor shell structures for acoustoelectric devices.

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